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惑星圏研究会

Influences of inner core radius on thermal convection in a rotating spherical shell near the critical Rayleigh number

数値実験に基づく内核半径変化が臨界レイリー数付近での回転球殻対流に与える影響

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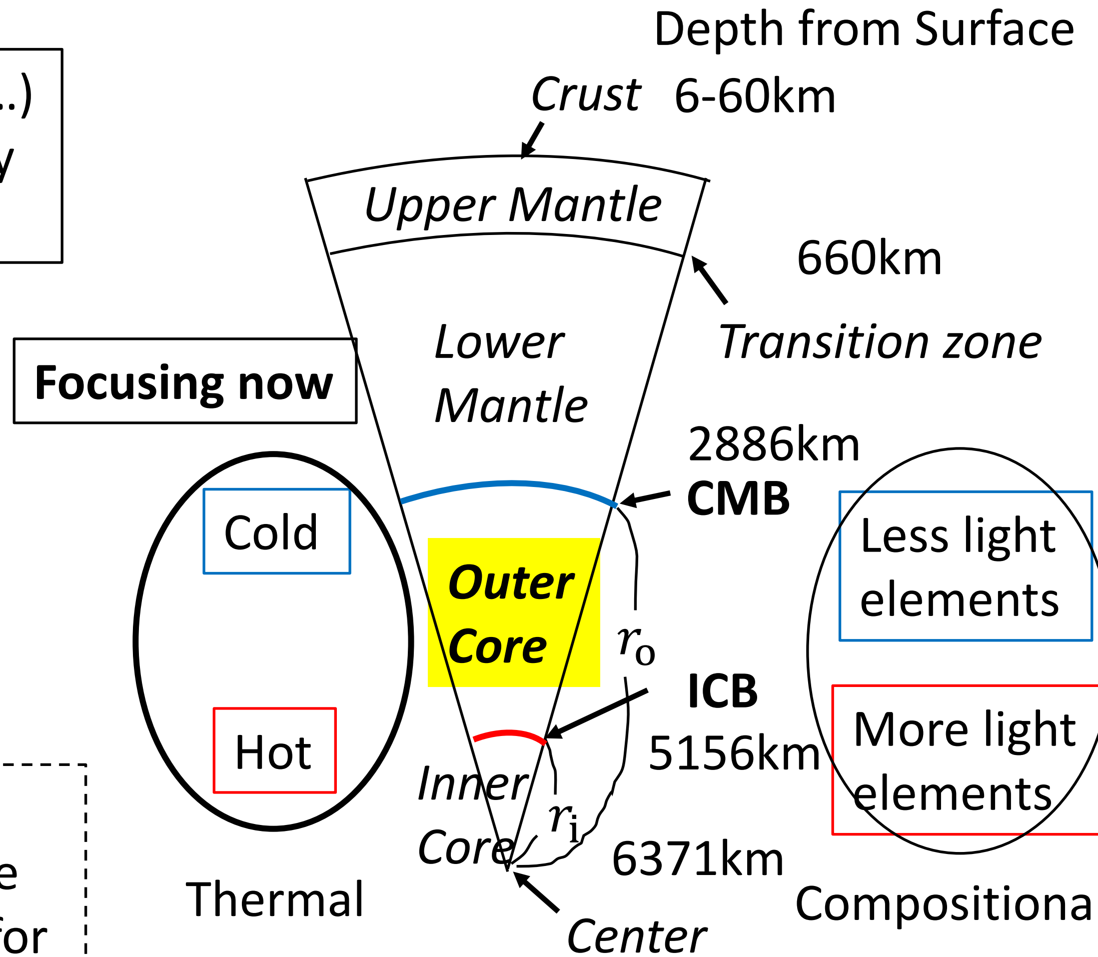
1.1 Focusing on the inner core radii

Convection of fluid Fe (+Si,O,Mg,…)
Kinetic energy → magnetic energy
= "Dynamo process"

Paleomagnetic analysis shows geomagnetic field has been sustained for at least **3.5 billion years**.

+

From **thermochemical evolution** calculation, the inner core could be growing from $\boldsymbol{r_\mathrm{i}/r_\mathrm{o} = 0}$ **to** $\boldsymbol{0.35}$ for the last billion years.



The present radius ratio is $r_\mathrm{i}/r_\mathrm{o} = 0.35$

--> It is important to investigate convection with various inner core radii.

1.2 Governing equations for numerical dynamo

Momentum equation

$$\frac{\partial \boldsymbol{u}}{\partial t} + (\boldsymbol{\nabla}\times\boldsymbol{u})\times\boldsymbol{u} = -\boldsymbol{\nabla}\left(P + \frac{1}{2}\boldsymbol{u}^2\right) - \underbrace{\boldsymbol{\nabla}\times(\boldsymbol{\nabla}\times\boldsymbol{u})}_{\text{Viscosity}} + \underbrace{\frac{Ra}{Pr}T\frac{\boldsymbol{r}}{r_o}}_{\textbf{Buoyancy}} - \underbrace{\frac{2}{E}\boldsymbol{e}_z\times\boldsymbol{u}}_{\text{Coriolis}} + \underbrace{\frac{1}{Pm\cdot E}(\boldsymbol{\nabla}\times\boldsymbol{B})\times\boldsymbol{B}}_{\text{Lorentz}}$$

Heat equation $\frac{\partial T}{\partial t} = -(\boldsymbol{u}\cdot\boldsymbol{\nabla})T + \frac{1}{Pr}\boldsymbol{\nabla}^2 T$

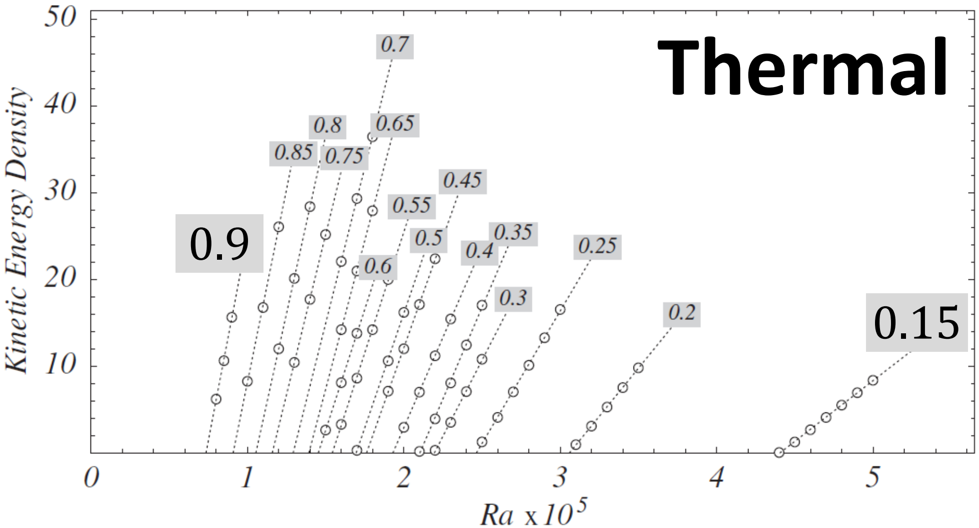
Continuity equation of incompressible fluid $\boldsymbol{\nabla}\cdot\boldsymbol{u} = 0$

Magnetic induction equation $\frac{\partial \boldsymbol{B}}{\partial t} = \boldsymbol{\nabla}\times(\boldsymbol{u}\times\boldsymbol{B}) - \frac{1}{Pm}\boldsymbol{\nabla}\times(\boldsymbol{\nabla}\times\boldsymbol{B})$

Gauss's law for magnetic field $\boldsymbol{\nabla}\cdot\boldsymbol{B} = 0$

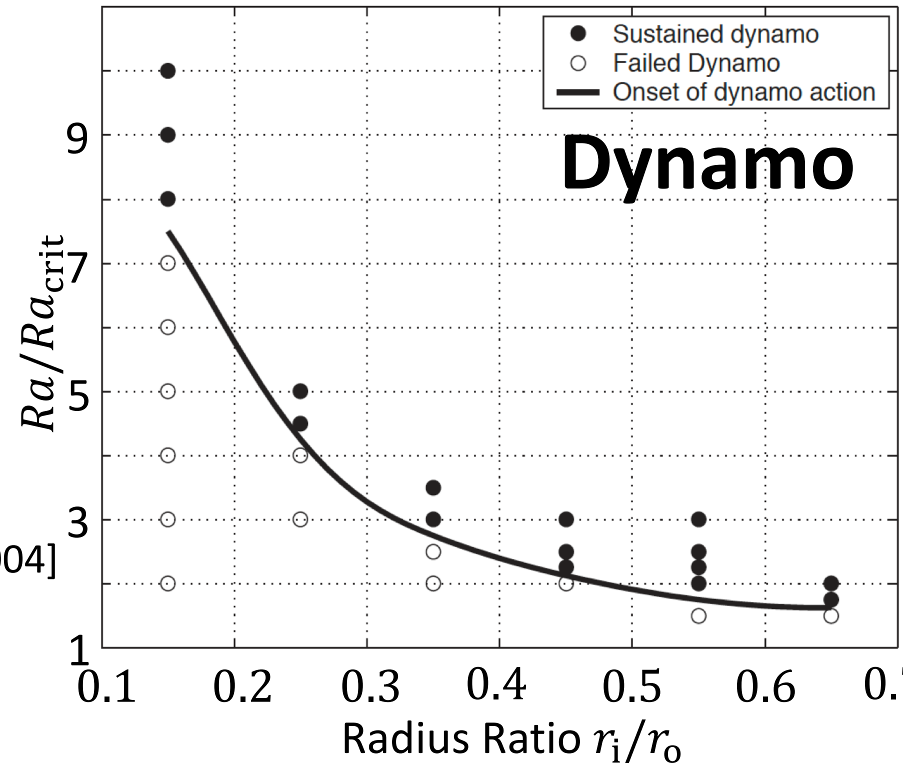
Name	Symbol	Definition	Meaning	Estimation
Rayleigh number	Ra	$\alpha_T g_0(\Delta T)L^3/\nu\kappa_T$	Buoyancy strength	10^{28}
Ekman number	E	$\nu/\Omega L^2$	Rotation weakness	10^{-15}
Prandtl number	Pr	ν/κ_T	Thermal diffusion weakness	0.1
Magnetic Prandtl number	Pm	ν/η	Magnetic diffusion weakness	10^{-6}

1.3 Previous studies



Onset of thermal convection [Al-Shamali et al., 2004]

$r_\mathrm{i}/r_\mathrm{o}$: smaller → Ra_crit:bigger



Dynamo onset [Heimpel et al., 2005]

Rayleigh Number (related to buoyancy)

$$Ra = \frac{\alpha_T g_0 (\Delta T) L^3}{\nu \kappa_T}$$

*The critical Rayleigh number Ra_crit represents onset of thermal convection.

$r_\mathrm{i}/r_\mathrm{o}$: smaller
→ Ra needed to sustain dynamo: larger

- Influence of inner core radius on convection is not fully understood.

1.4 Purpose of this study

For fully understanding of the convection in the outer core at **the past Earth**, we investigate properties of dynamo action with the smaller inner core using numerical dynamo open code Calypso [Matsui et al., 2014].

- Calculation of kinetic/magnetic energy density
- Calculation of length scale of flow

⇩

- ➢ Maximum growth mode in magneto-convection model
- ➢ Influence of magnetic field on convection near dynamo onset

2.1 Code and initial/boundary condition

We used numerical dynamo open code Calypso [Matsui et al., 2014].

- radial derivatives … second order finite difference
- spectral method
  solenoidal vector field … poloidal and toroidal
- time stepping
  the linear diffusive terms ... the Crank-Nicolson
  the Coriolis force and the nonlinear terms ... second order Adams-Bashforth

Initial condition [cf. Benchmark case 1 proposed by Christensen et al. (2001)]

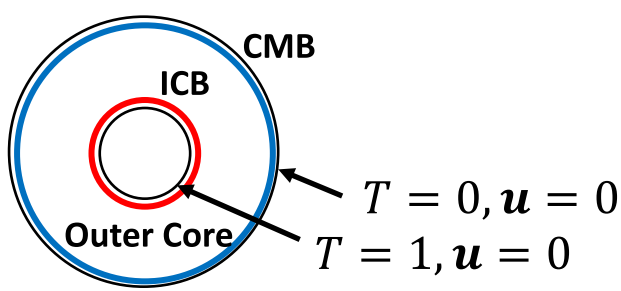
Given **various modes** of temperature perturbation by

$$T(r,\theta,\phi) = \sum_{l=1}^{l_{\max}} T_l^l(r) Y_l^l(\theta,\phi)$$

Given **an axial dipole** as a seed magnetic field;

$$\boldsymbol{B} = \boldsymbol{\nabla}\times\boldsymbol{\nabla}\times\left(B_{S1}^0(r)Y_1^0(\theta,\phi)\hat{r}\right) + \boldsymbol{\nabla}\times(B_{T2}^0(r)Y_2^0(\theta,\phi)\hat{r})$$

Boundary condition



Mantle and the inner core
…Electrically insulated

2.2 Parameters setting

	Case 1	Case 2	Case 3
r_i/r_o	0.15	0.25	0.35
$Ra_{crit}[\times10^5]$	1.09	0.72	0.56
$Ra[\times10^5]$	8.7~17	1.4~7.0	0.84~4.0
Ra/Ra_{crit}	7.0~15.6	1.9~9.7	1.5~7.1

<-- Thermal simulation

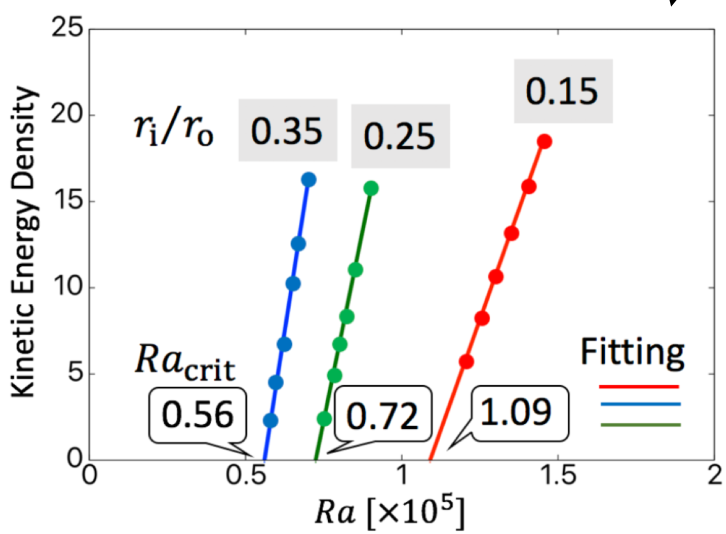
*The critical Rayleigh number Ra_{crit} represents onset of thermal convection.

$$E = 1\times10^{-3}, Pr = 1, Pm = 5$$

Kinetic/Magnetic energy density

$$E_{kin} = \frac{1}{2V_S}\int_{V_S} \boldsymbol{u}^2 dV$$

$$E_{mag} = \frac{1}{2V_S E Pm}\int_{V_S} \boldsymbol{B}^2 dV$$



3.1 Kinetic/magnetic energy density

Time average of E_{kin} and E_{mag} for $t/\tau_\eta = 1.5$ to 2.0

* $\tau_\eta (= L^2/\eta)$: magnetic diffusion time

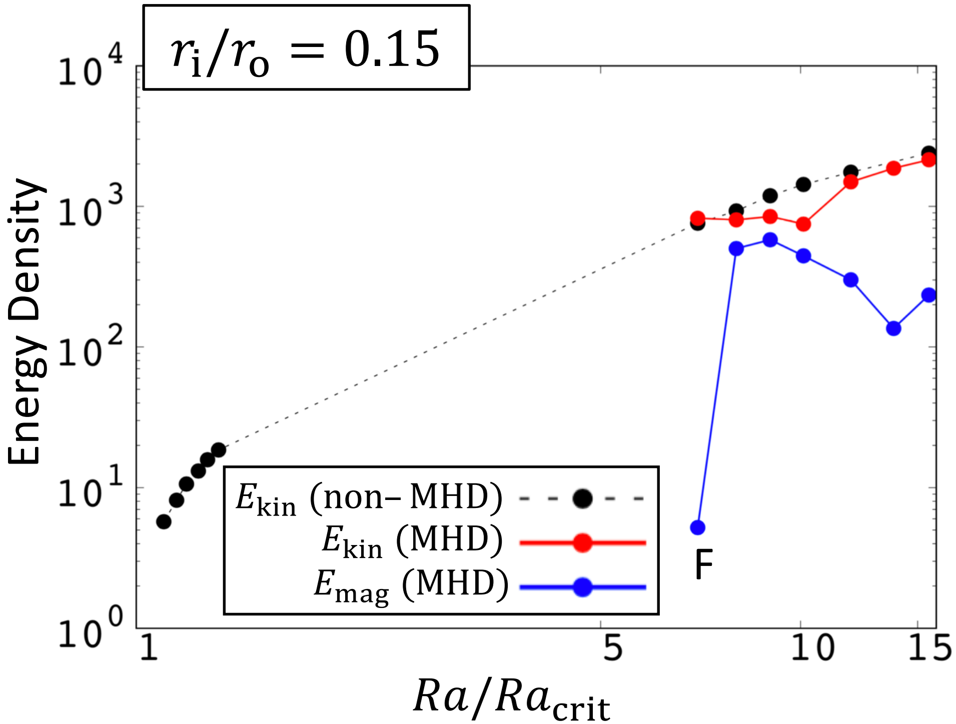
➢ Common point

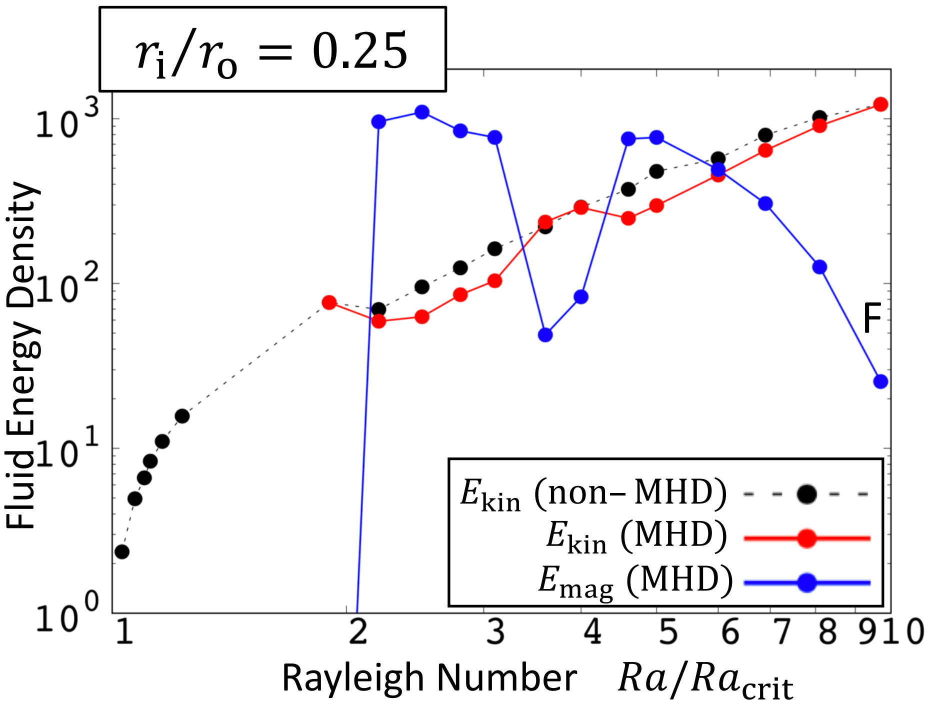
E_{kin} (MHD) $<$ E_{kin} (non–MHD) under large magnetic field.

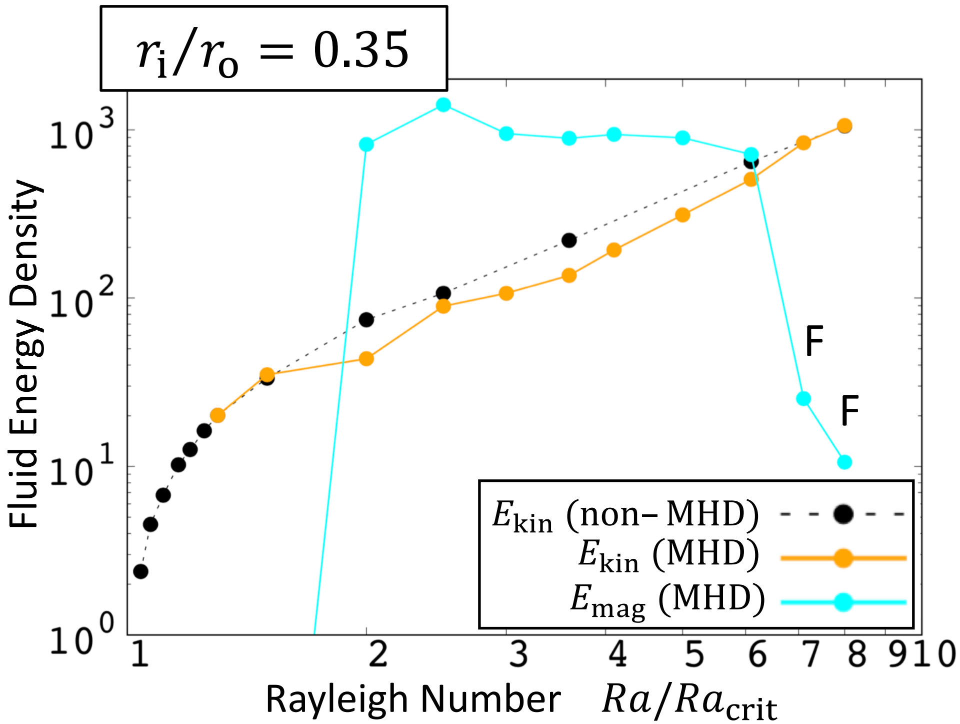
➢ Deferent points

- In $r_\mathrm{i}/r_\mathrm{o} = 0.15$, $E_{\mathrm{mag}} < E_{\mathrm{kin}}$
- In $r_\mathrm{i}/r_\mathrm{o} = 0.25$, $E_{\mathrm{mag}} > E_{\mathrm{kin}}$ or $E_{\mathrm{mag}} < E_{\mathrm{kin}}$
- In $r_\mathrm{i}/r_\mathrm{o} = 0.35$, $E_{\mathrm{mag}} > E_{\mathrm{kin}}$

Lorentz force is strong for $E_{\mathrm{mag}} > E_{\mathrm{kin}}$.

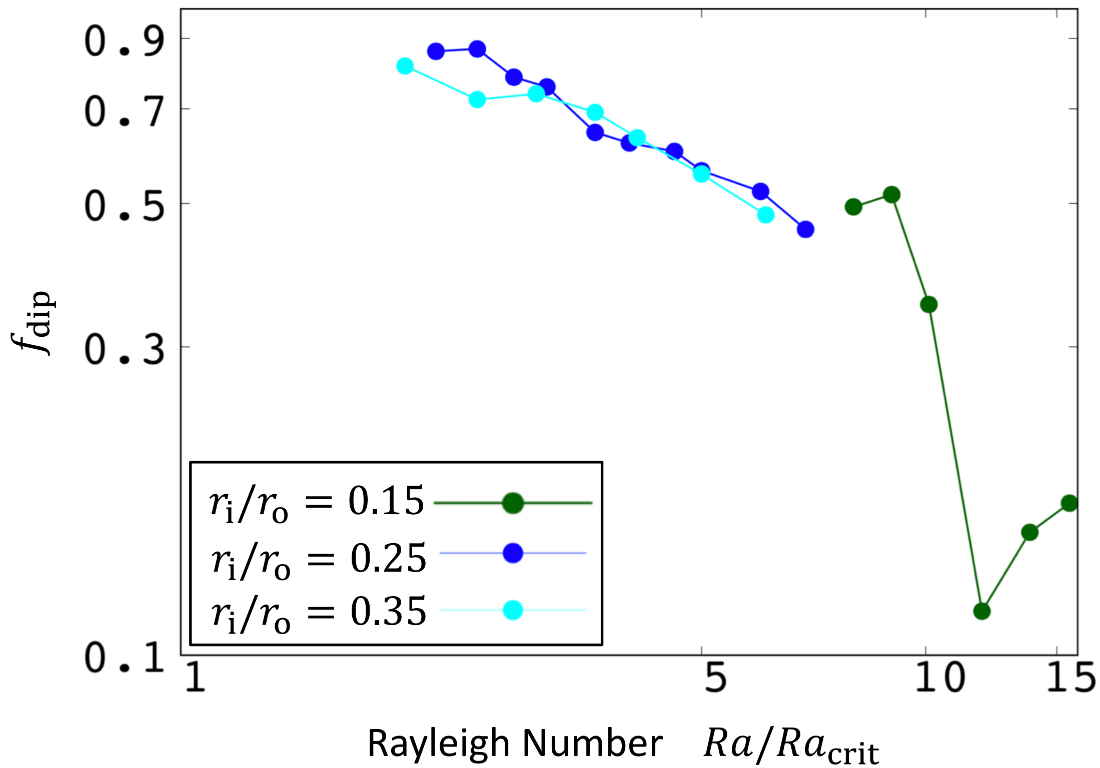






- E_{mag} drops at $Ra/Ra_{crit} = 3.1$ and 3.6 in $r_i/r_o = 0.25$.
- E_{mag} is comparable in $r_i/r_o = 0.35$.

3.2 Dipolarity

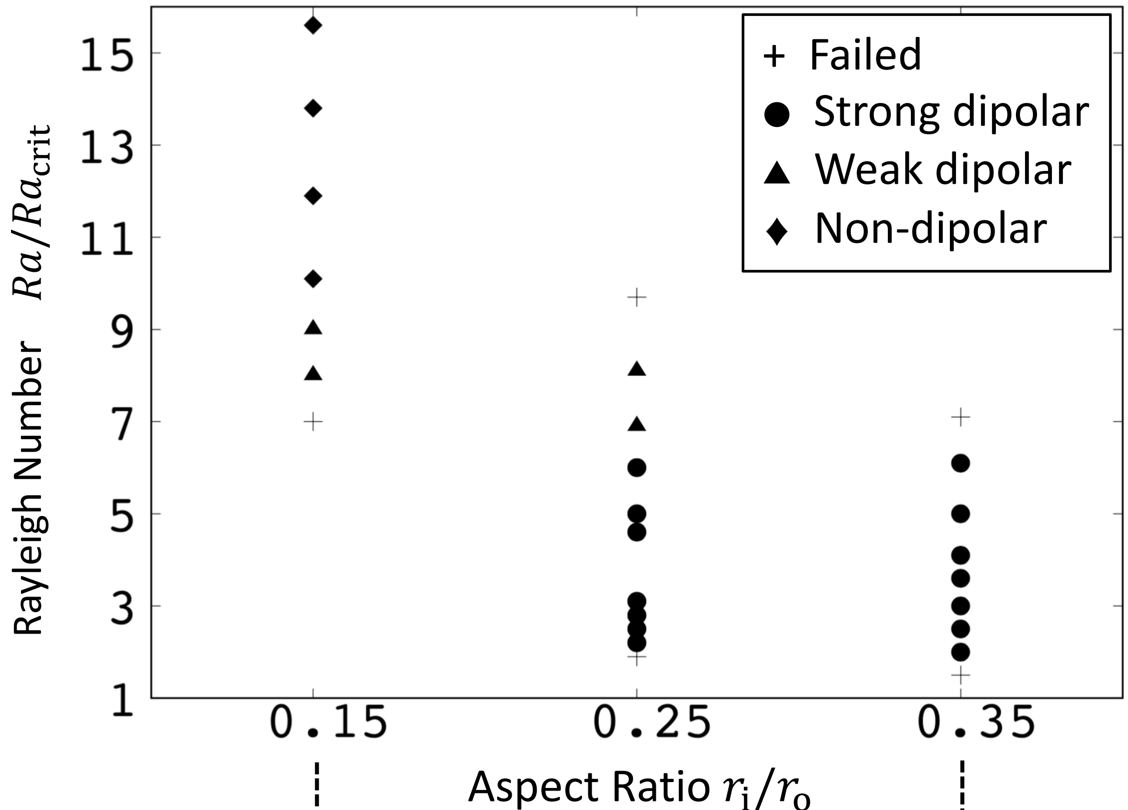


Dipolarity is an index which represents strength of an axial dipole component.

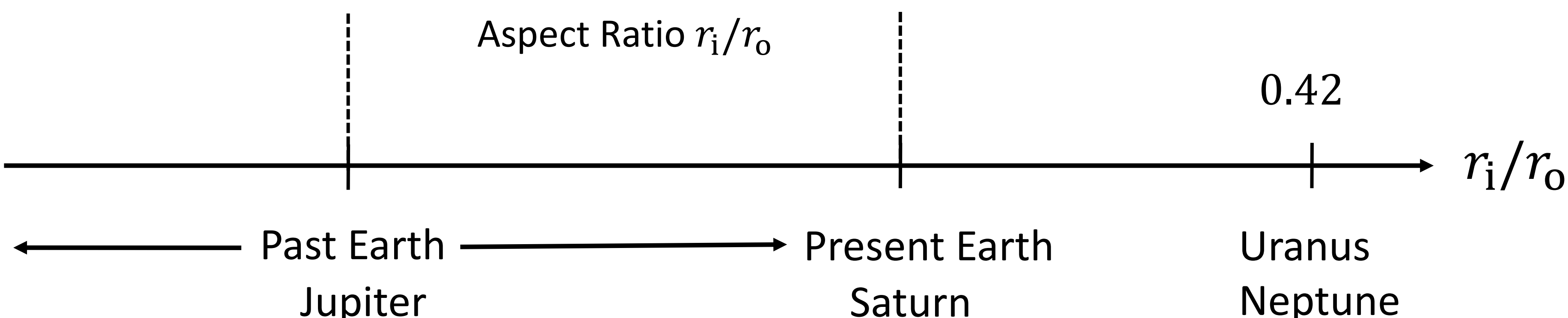
$$f_{\rm dip} = \left(\frac{E_{\rm mag}^{(l=1,m=0)}(r=r_{\rm o})}{\sum_{l=1}^{l_{\max}} \sum_{m=0}^{l} E_{\rm mag}^{(l,m)}(r=r_{\rm o})} \right)^{1/2}$$

l: Spherical harmonic degree
m: Spherical harmonic order

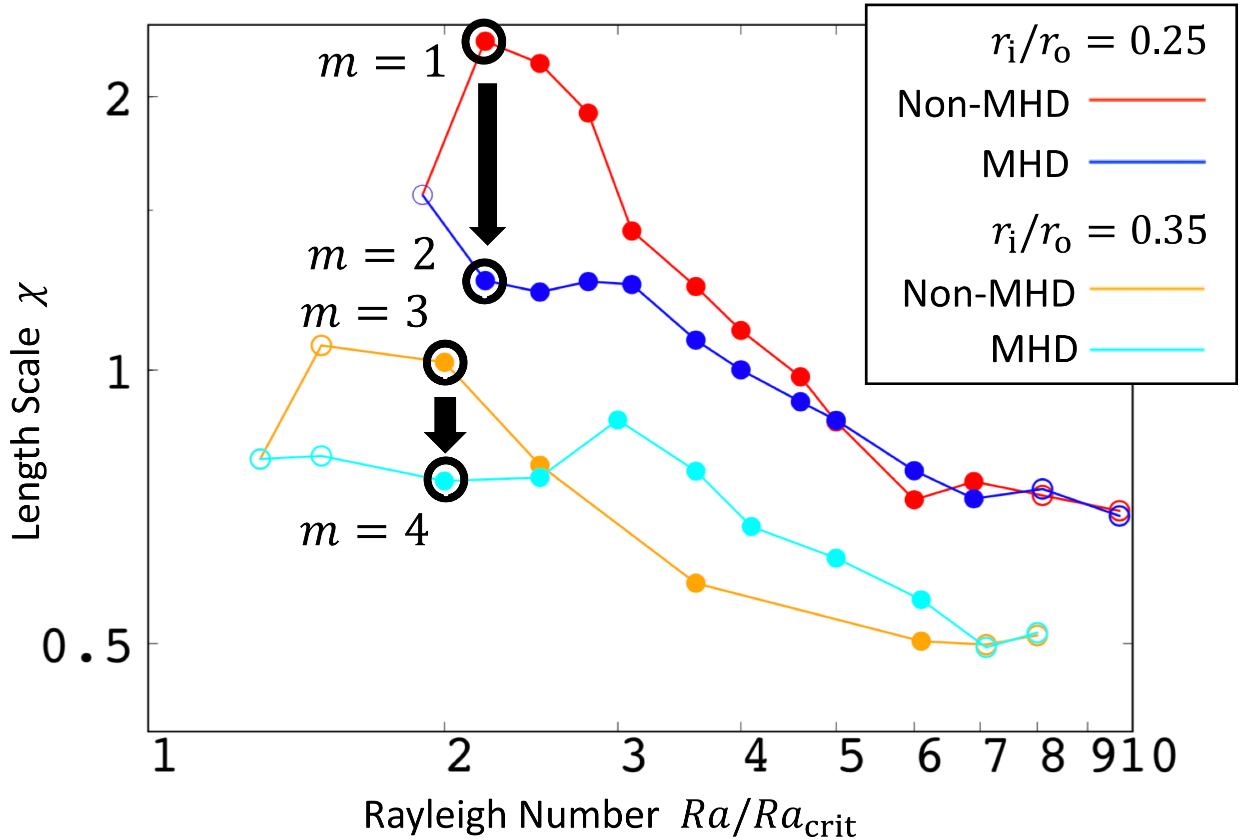
3.3 Dynamo regime in various aspect ratios



- When Ra becomes large, an axial dipolar component becomes small.
- In the smaller inner core size cases, non-dipolar components become large.



4.1 Length scale of flow in azimuthal direction



$$\chi = \frac{\overline{\pi < \boldsymbol{u}^2 >}}{\sum m < \boldsymbol{u}_m^2 >} L = \left\{ \pi \Big/ \left(\sum m \frac{\overline{< \boldsymbol{u}_m^2 >}}{< \boldsymbol{u}^2 >} \right) \right\} L$$ [cf. King and Buffett, 2013]

(m: Spherical harmonic order, $\boldsymbol{u}$: velosity, L: the outer core thickness)

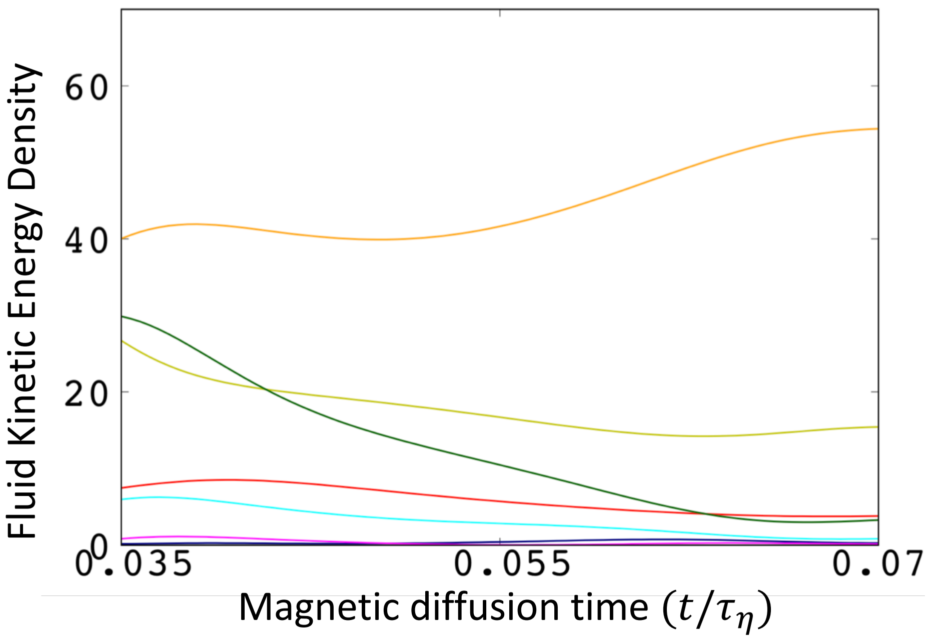
4.2 Growth in different modes

Simulation using linear terms (under background magnetic field and $\partial \boldsymbol{B}/\partial t = 0$)

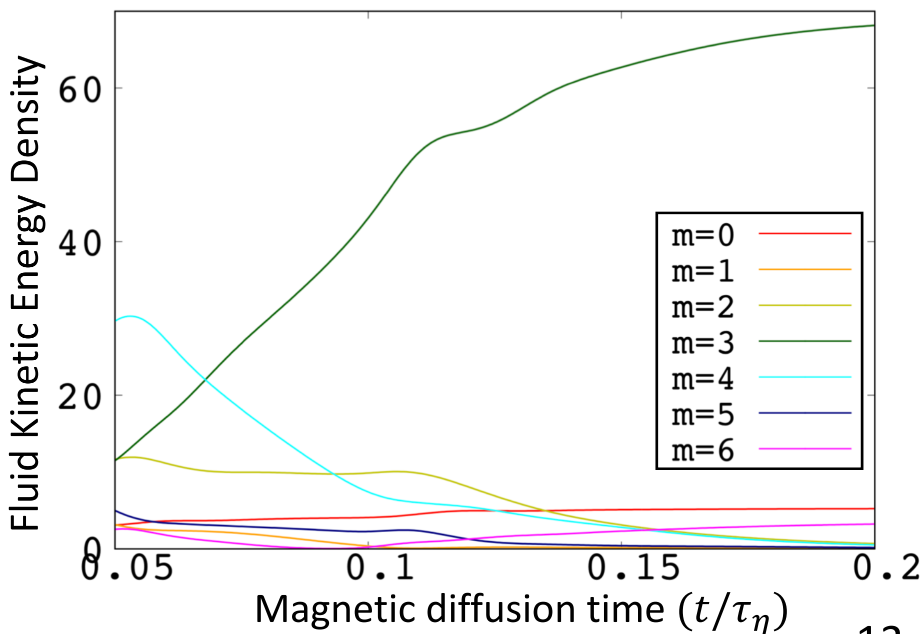
$$\frac{\partial \boldsymbol{u}}{\partial t} + (\boldsymbol{\nabla}\times\boldsymbol{u})\times\boldsymbol{u} = -\boldsymbol{\nabla}\left(P + \frac{1}{2}\boldsymbol{u}^2\right) - \boldsymbol{\nabla}\times(\boldsymbol{\nabla}\times\boldsymbol{u}) + \frac{Ra}{Pr}T\frac{\boldsymbol{r}}{r_\mathrm{o}} - \frac{2}{E}\boldsymbol{e}_z\times\boldsymbol{u} + \frac{1}{Pm\cdot E}(\boldsymbol{\nabla}\times\boldsymbol{B})\times\boldsymbol{B}$$

$$\frac{\partial T}{\partial t} = -(\boldsymbol{u}\cdot\boldsymbol{\nabla})T + \frac{1}{Pr}\boldsymbol{\nabla}^2 T \qquad \boldsymbol{\nabla}\cdot\boldsymbol{u} = 0$$

$r_\mathrm{i}/r_\mathrm{o} = 0.25$, $Ra/Ra_\mathrm{crit} = 2.2$



$r_\mathrm{i}/r_\mathrm{o} = 0.35$, $Ra/Ra_\mathrm{crit} = 2.0$



4.3 Maximum growth mode

Maximum growth mode near dynamo onset is

- $m = 1$ (no B) → ~~$m = 1$~~ (with stable B) → $m = 2$ (with B) in $r_\mathrm{i}/r_\mathrm{o} = 0.25$
- $m = 3$ (no B) → ~~$m = 3$~~ (with stable B) → $m = 4$ (with B) in $r_\mathrm{i}/r_\mathrm{o} = 0.35$

Estimated growth rate p from the time history of E_kin which is set as $\exp(pt)$

m	0	1	2	3	4	5	6
$p\ (r_\mathrm{i}/r_\mathrm{o} = 0.25)$	-32.41	68.30	-84.09	-93.97	-40.93	-90.08	-448.8
$p\ (r_\mathrm{i}/r_\mathrm{o} = 0.35)$	7.62	-2.96	-24.91	27.03	-22.48	-27.47	13.47

5 Summary and future work

- In the smaller inner core size cases, non-dipolar components become large.
- Length scale of flow in MHD cases is smaller than that in non-MHD cases with small Ra.
- Maximum growth mode in MHD cases is larger than that in magneto-convection model.

✓ Future works

- More simulations in some Ra in $r_\mathrm{i}/r_\mathrm{o} = 0.25, 0.35$

→ Investigation of maximum growth mode dependency on Ra and radius ratio and initial field

- Derivation of the maximum growth mode with/without the inner core from liner stability analysis… scale of Ra_crit on Ekman number and aspect ratio

[Bissopp, 1958; Chandrasekhar, 1961; Roberts, 1968; Busse, 1970; Jones et al., 2000]